What can be verified locally?

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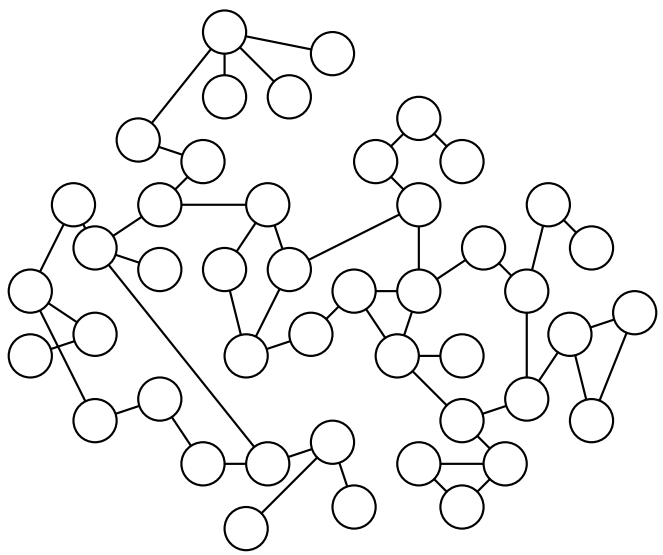
Goal

- Classify problems in the distributed context, according to their difficulty.
- Build a hierarchy of complexity classes in the context of the LOCAL model.

Local Model

- The distributed network is represented by a graph.



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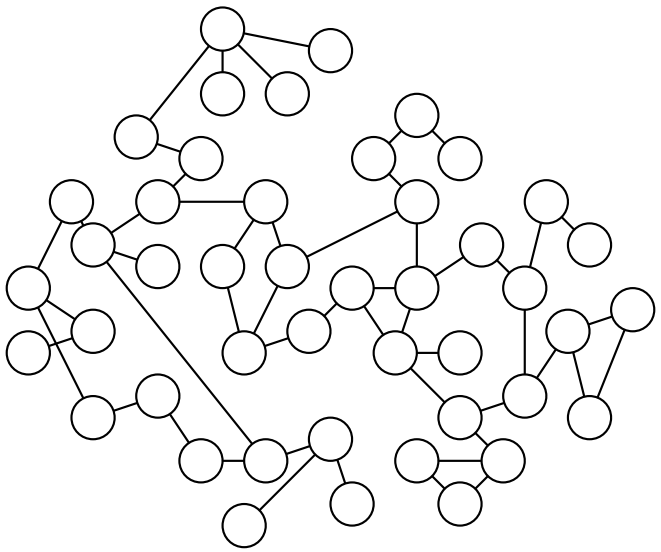

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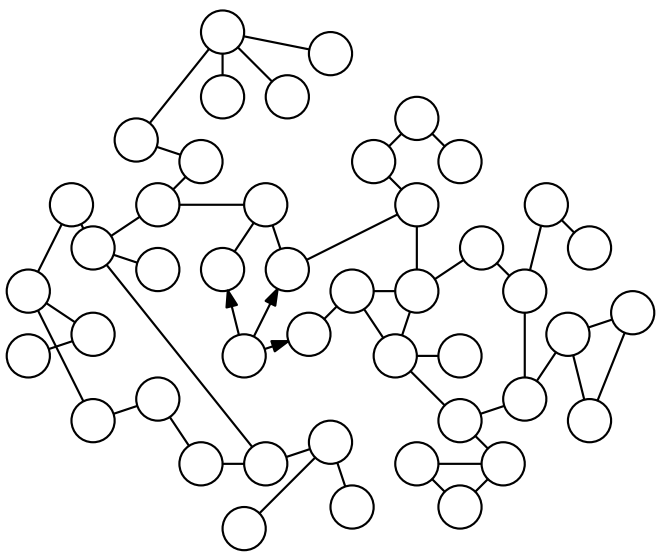

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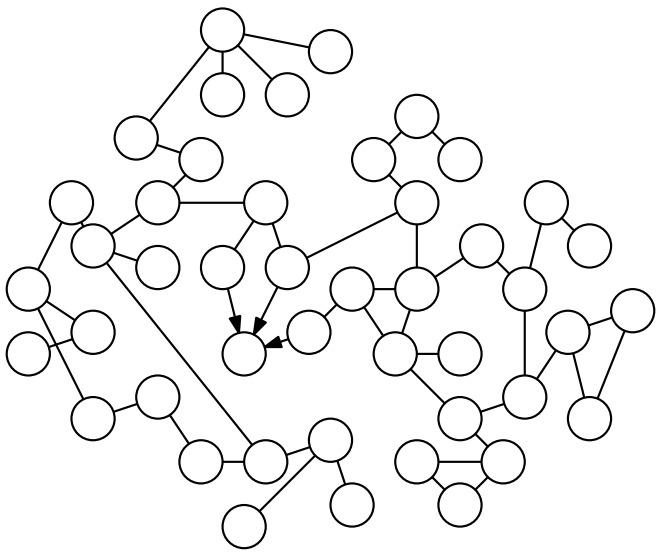

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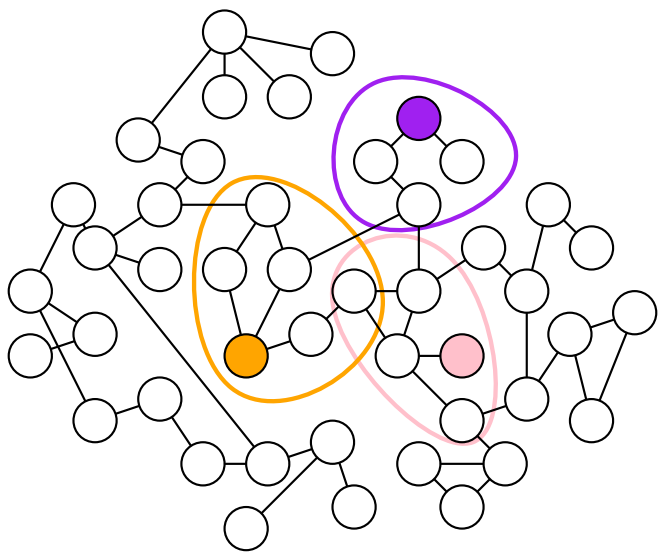

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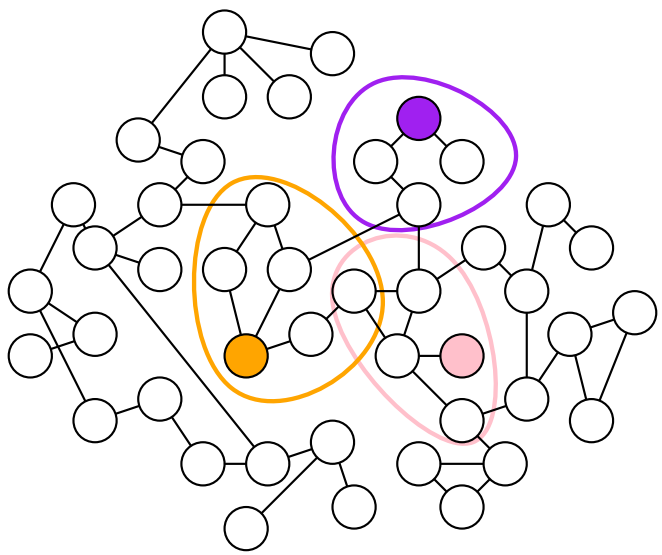

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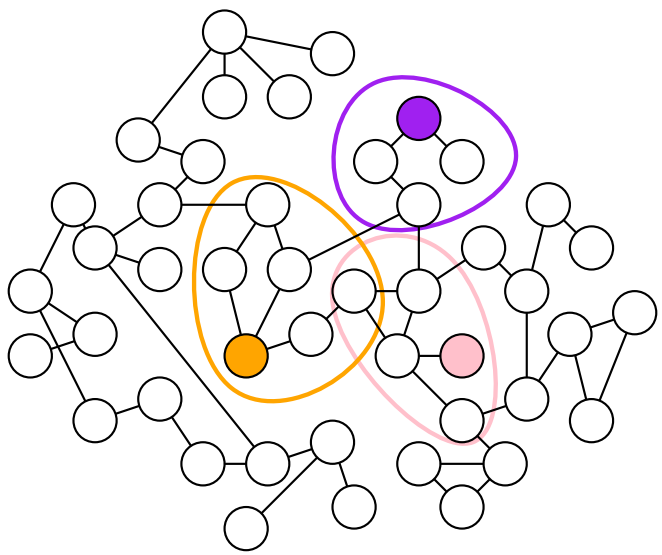
- We want t to be constant.



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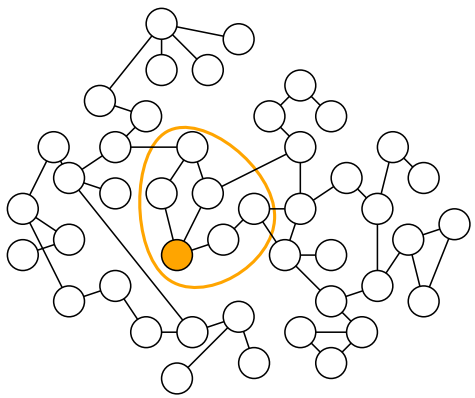

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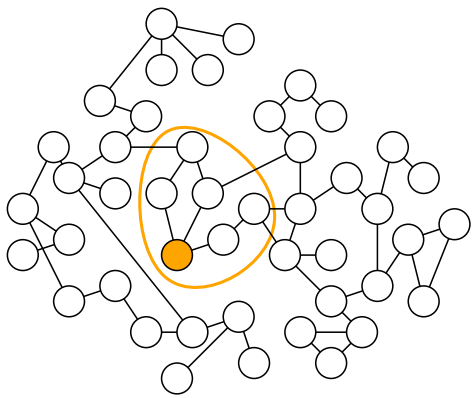

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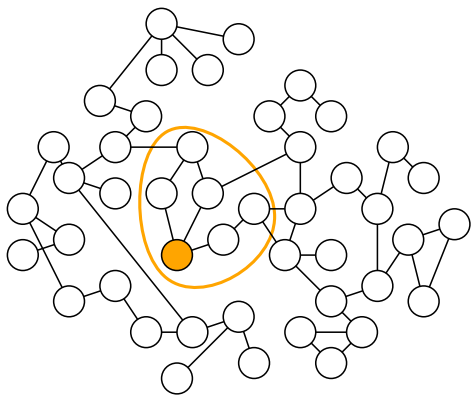

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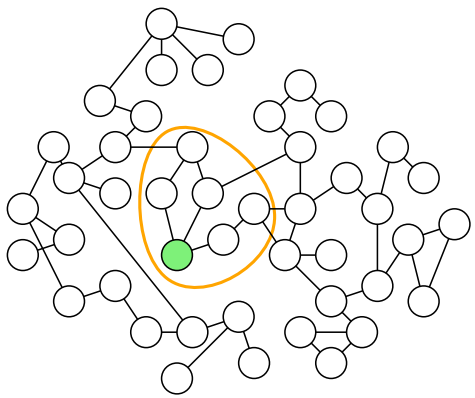

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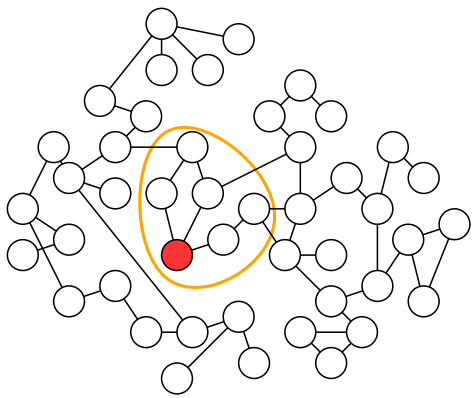

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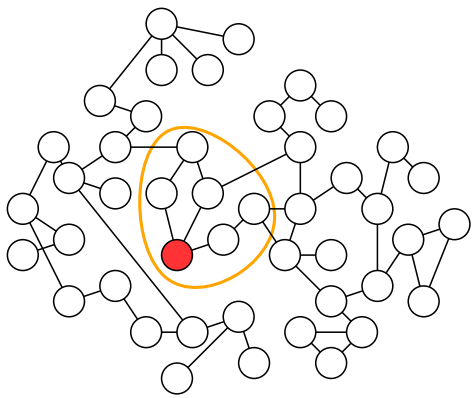

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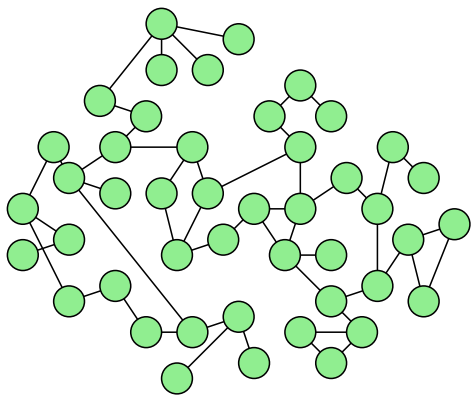

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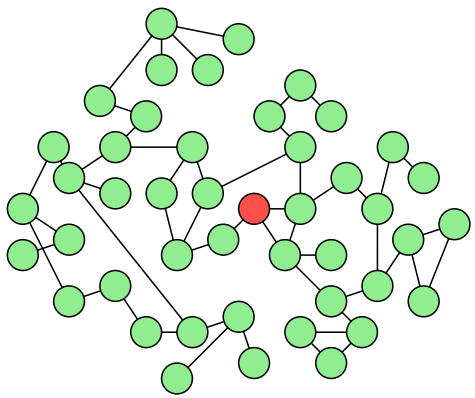

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Example: Proper Coloring

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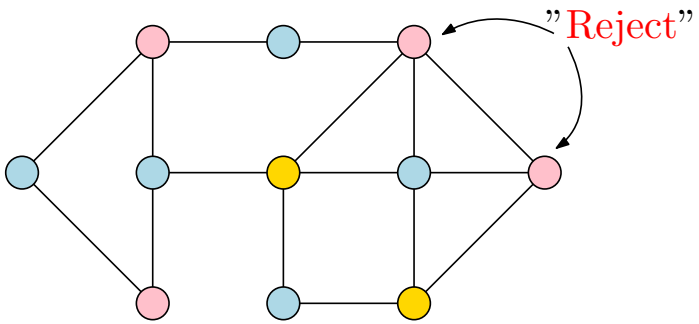

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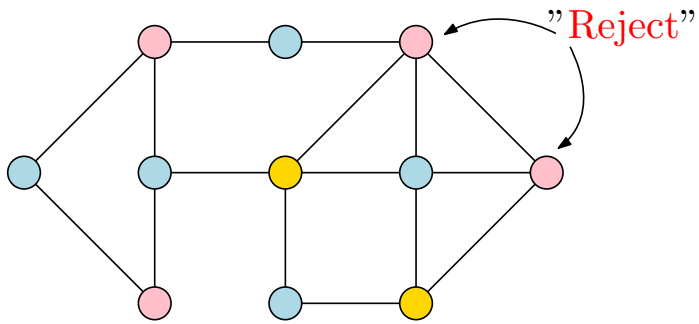

- *Local Decision (LD)* is the class of distributed languages that can be locally decided [NS '95].

Distributed vs Centralized

Definition of LD

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Definition of P

$L \in \mathrm{P}$ if there is a polynomial time algorithm A such that,

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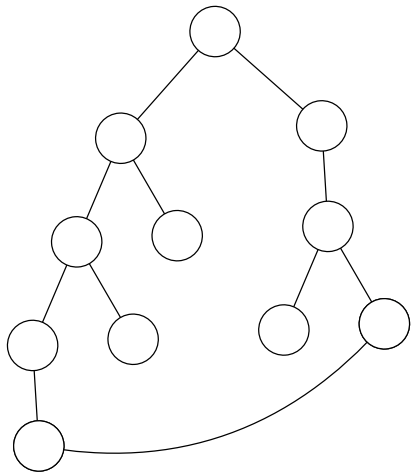

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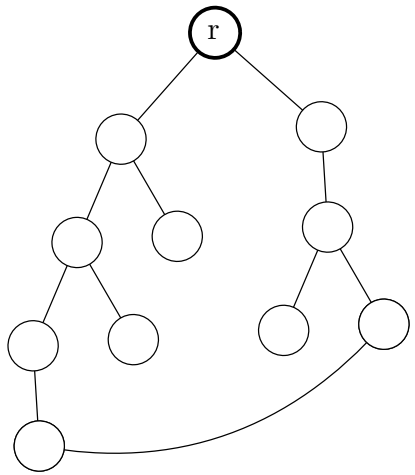

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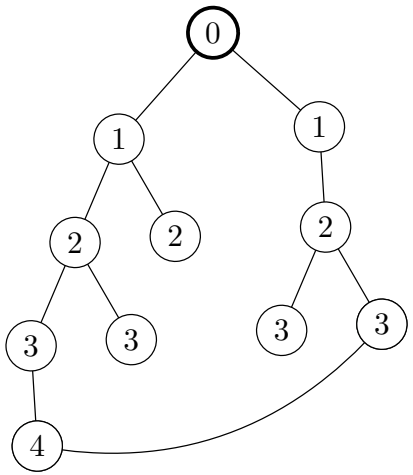

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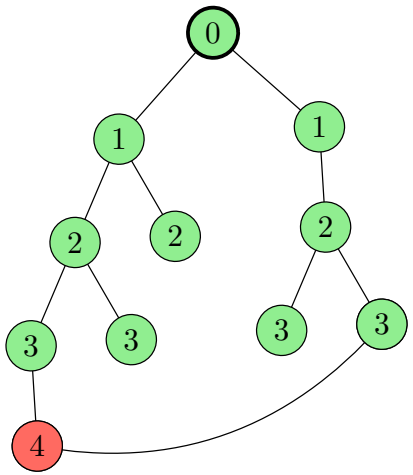

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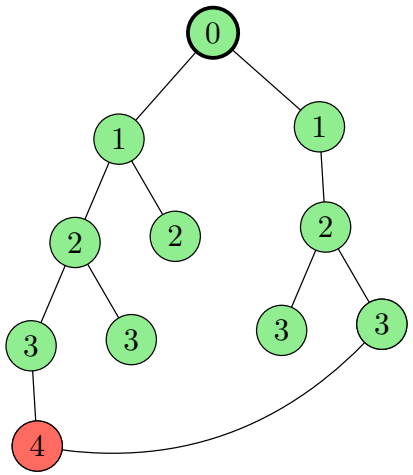

- *Nondeterministic LD (NLD)* is the class of distributed languages that can be locally verified [FKP '11].

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Definition of NP

$L \in$ NP if there is a polynomial time algorithm A such that,

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Goal

- Build a hierarchy of complexity classes in the distributed setting.
- Distributed hierarchies in other setting:
  - [Reiter '14] in the context of automata;
  - [FFH '16] in a model inspired by the CONGEST one.

Complexity Classes

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Π_2^{loc} Class

- Π_2 class: An input instance satisfies a certain property in Π_2 iff

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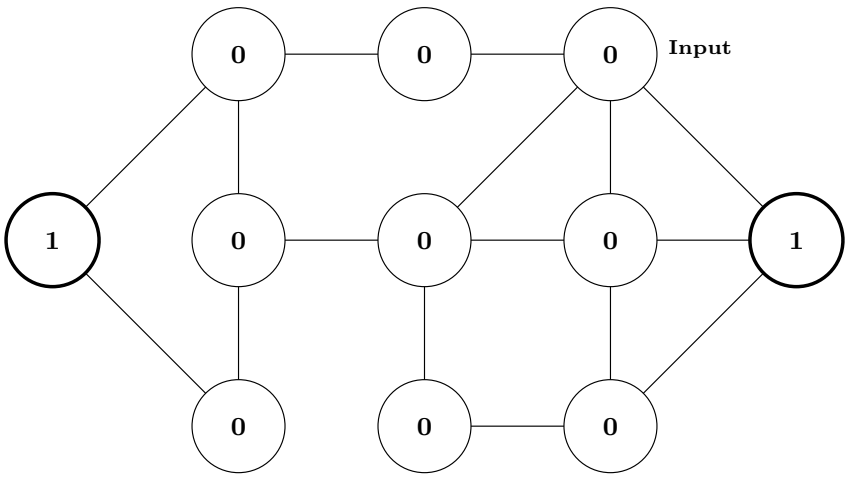
- Two party game between a *disprover* and a *prover*.

Exactly Two Selected



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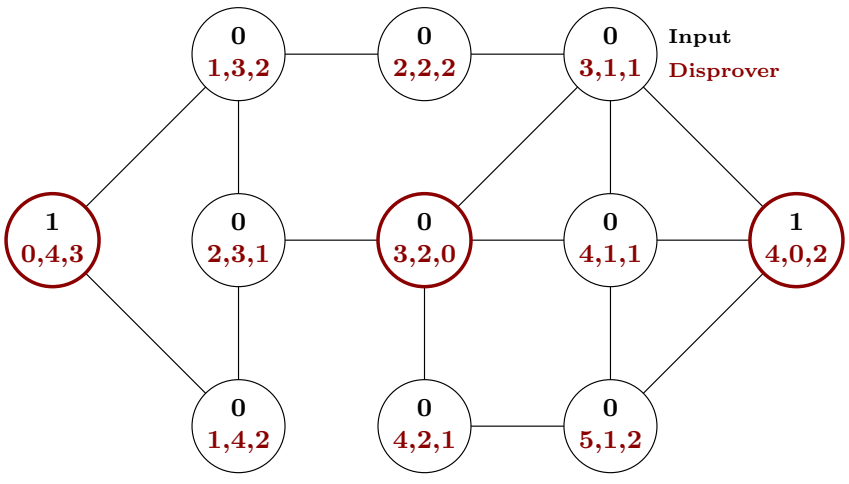

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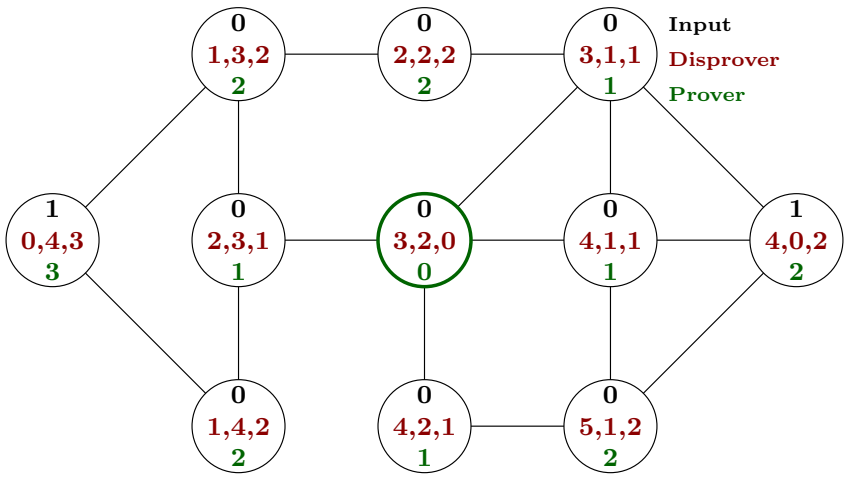

Π_1^{loc}: The Role of the Last Universal Quantifier

- Π_1^{loc}:

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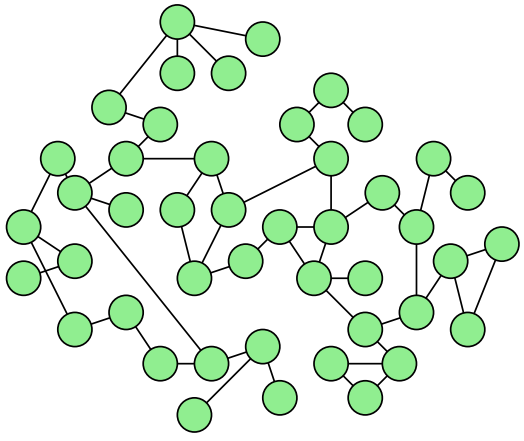
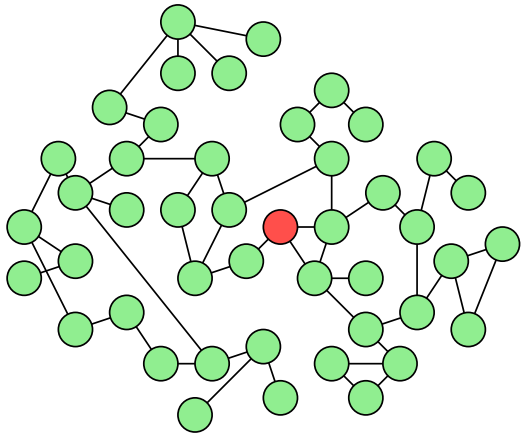
- Problems that can be solved only if a specific node knows (an upper bound of) the size of the network!

Complementary Classes

In a class:



A globaly accepted input instance.



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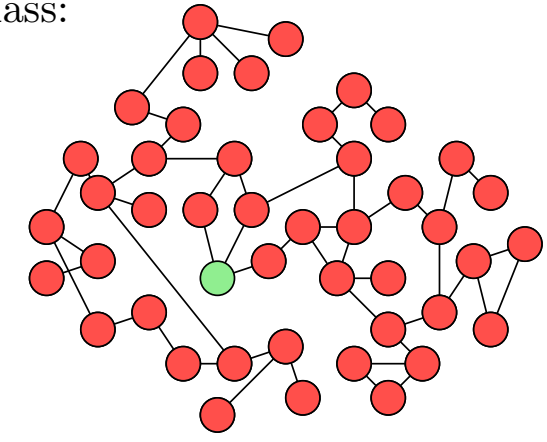

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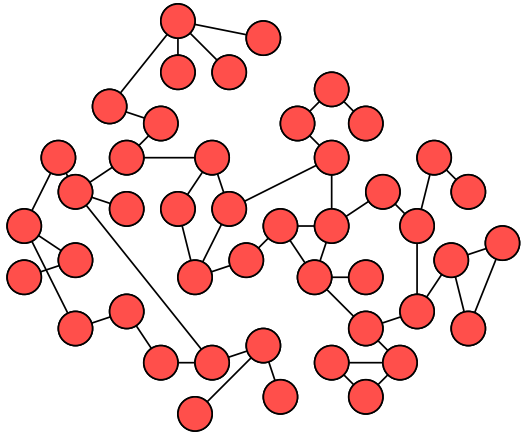

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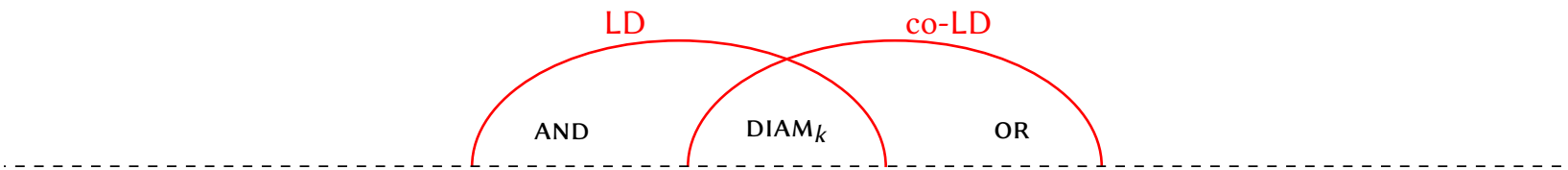
Local Hierarchy



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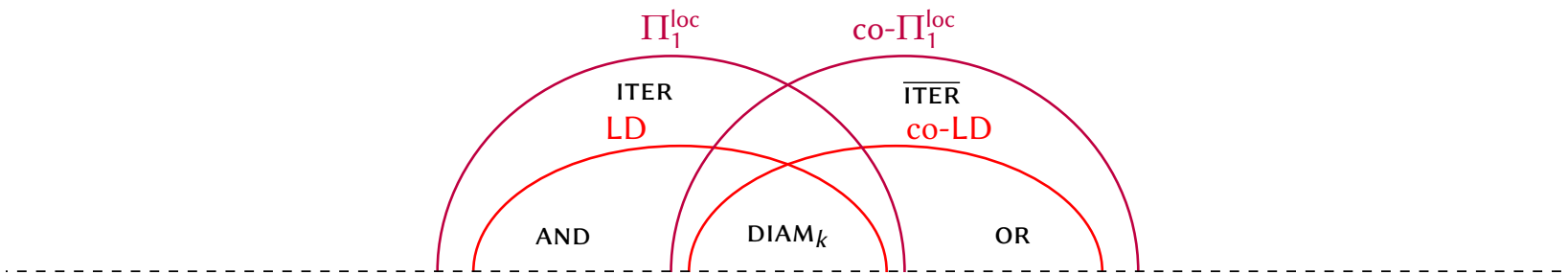

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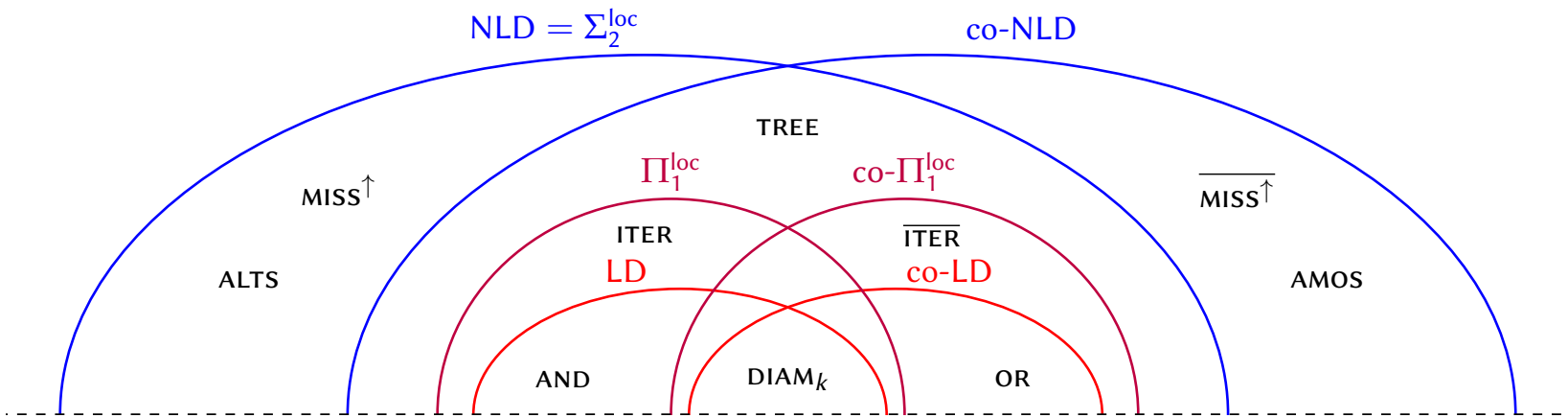

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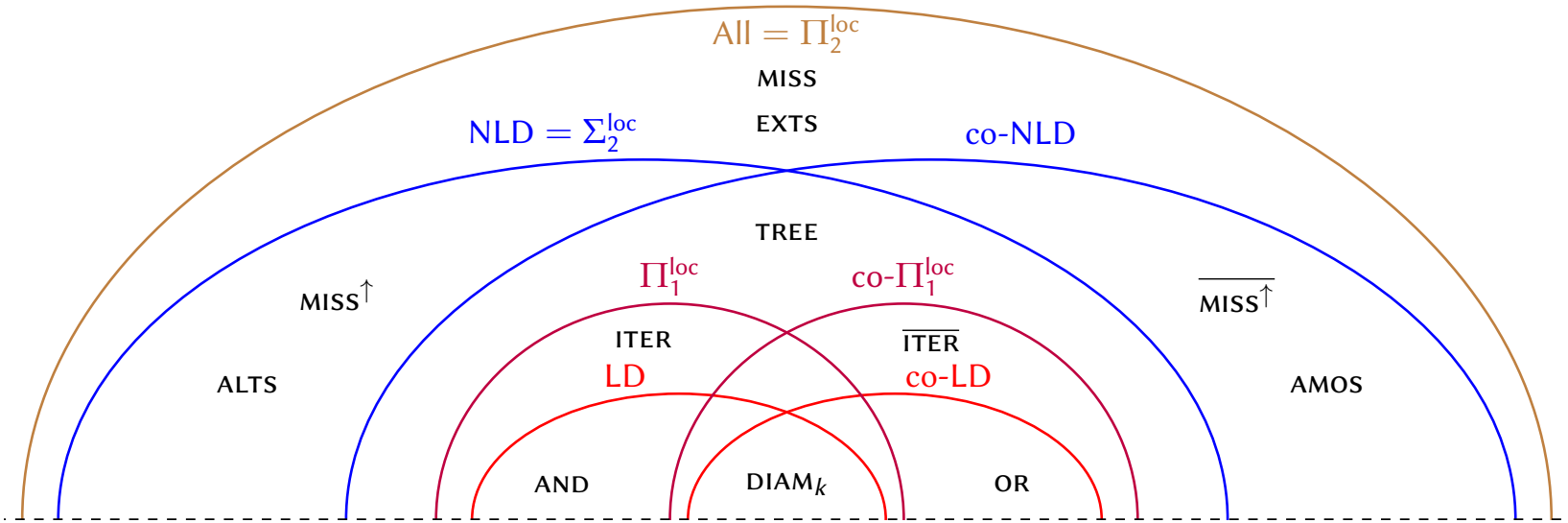

$$\text{LD} \subset \Pi_1^{loc} \subset \text{NLD} = \Sigma_2^{loc} \subset \Pi_2^{loc} = \text{All (all inclusions are strict).}$$

Open Problems

- Unbounded size id-independent certificates:
  - find a complete problem for Π_1^{loc} and co-Π_1^{loc};
  - find a problem in the intersection between the classes Π_1^{loc} and co-Π_1^{loc}.
- Bounded size ($O(\log n)$) id-dependent certificates [FFH '16]
  - we don't know if the hierarchy collapses;
  - there are no separating problems for Σ_2^{loc} and Σ_3^{loc} (neither for classes higher in the hierarchy).

Thank you!